# An Extract of some Physico-Mathematical Discourses contained in Mr. COTES's Hydrostatical and Pneumatical Lectures: printed for the use of those that go the Course of Experiments.

*The exact estimate of all manner of Pressures. The invention of the Center of Pressure upon any proposed Plane reduced to the Problem of finding the Center of percussion.*

WE are now to determine the quantity of that Pressure which any surface sustains that is expos'd to the Gravitation of a Fluid, this must be done gradually beginning with those Cases which are most simple and easy, and afterwards proceeding to those which are more complex and difficult. Let a Vessel *ABCD* ( Fig. 1. ) be propos'd containing any Fluid, suppose Water, let *AB* be the upper surface of the Water, and *CD* the Bottom of the Vessel ; the pressure upon any part of that Bottom suppose *GH* will be equivalent to the weight of a Column of Water *GHIK*, having the part *GH* for its base and *GI* or *HK* the depth under Water for its altitude. This seems to be self evident and may best be prov'd by the absurdity of any contrary supposition; for if it be said that *GH* sustains a greater weight than that of the Column *GHIK*, the excess must come from the adjoyning Columns *ACGI* and *KHDB*, now for the like reason it ought to be said that *CG* sustains a greater weight than that of the Column *ACGI*, and *HD* a greater than that of the Column *KHDB*; but if this were true, then would all the parts *CG*, *GH* and *HD* together, of that whole plane *CD* sustain a greater weight than that of the Columns together, or the whole Water which is above it namely *ACDB*, which is absurd. The like absurdity will follow if it be said that *GH* sustains a lesser pressure than the weight of the Column *GHIK*; the weight of that Column then, which is perpendicularly incumbent upon it is exactly equivalent to the pressure which it sustains. This is the Quantity of pressure upon the Plane *GI* in the case that has already been describ'd. If the Figure of the Vessel be any way alter'd, the pressure will still be the same if the perpendicular distance of the Plane *GH* from the upper surface of the Water contain'd in the Vessel of what ever Figure it be, remain unalter'd. Thus (in Fig. 2. 3.) if *LNGHOM* be a Vessel of any irregular Figure, *LM* be the upper surface of the Water, and the Perpendicular distance of *GH* below *LM* namely *GI* or *HK* be the same as before ; the pressure of the Vesselled Water *LNGHOM* upon the Bottom *GH* will be equal to the same weight of the Column *GHIK* as before, though the Vessell'd Water *LGHM* be much

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lefs than that Column as in (Fig. 2.) or much greater as in (Fig. 3.) The preffure is not to be eftimated by the quantity of Water but by its Altitude. For if the quantity of Water *LGHM* (Fig. 2.) be a thoufand times lefs than *IGHK* as we may eafily fuppofe it to be, and the quantity of Water *LGHM* (Fig. 3.) be a thoufand times greater than the fame *IGHK*, then the quantity of this latter will be a million of times greater than the former, neverthelefs both will equally prefs upon their bottoms *GH*, namely with a force equivalent to the weight of the perpendicular Column *GHIK*, which may defervedly be accounted a paradox in Hydroftaticks, but may thus (among other ways) be render'd intelligible. Let us conceive each of thofe Veffels plac'd in a larger *ABCD*, the preffure upon *GH* will be the fame whether we fuppofe the Water *LNGHOM* to be contain'd in its proper Veffel *LNGHOM* or imagining that Veffel to be away, we fuppofe its place to be fupplyed by the ambient Water *ACGNL* and *HOMBD*: for any parcel of Water may be conceiv'd to be kept in by the reft of the Water, which every way furrounds it as in a Veffel, fuppofing all at reft; now in this latter Cafe where we make the ambient Water *ACGNL* and *HOMBD* to be a Veffel to the Water *LNGHOM* the preffure upon *GH* is equivalent to the weight of the Column *GHIK* as has been already made out, therefore in the former where the Water *LNGHOM* was contain'd in its proper Veffel, the preffure upon *GH* will be alfo equivalent to the weight of the fame Column *GHIK*. By the fame way of reafoning we may conclude that the Water contain'd in any other more irregular Veffel as *LNGHOM* (Fig. 4.) preffes upon the bottom with a force equivalent to the weight of the Column of Water *GHIK*, having the faid bottom for its Bafis, & *GI* or *HK* the perpendicular diftance of the Plains *GH* and *LM* for its Altitude. If the Plane *GH* be oblique to the Horizon as in (Fig. 5.) The preffure upon *GH* from the Water of the Veffel *LNGHOM*, or from that of the Veffel *EGHF*, or from that of the larger Veffel *ACDB* will ftill be the fame, if the upper Surfaces *LM*, *EF* and *AB* be in the fame Plane or at the fame Altitude above *GH*. The Altitude is every where the meafure of Preffure whatever be the quantity of the Fluid, or however the containing Veffel be figur'd.

I fhould now proceed to eftimate the preffure upon Planes which are either perpendicular or oblique to the Horizon, but becaufe the feveral indefinitely fmall Parts of which fuch Planes are compofed are acted upon with different forces accordingly as the particles of Water, by which they are immediately touch'd, happen to be at different depths, and fince the total preffure is made up of all thefe different forces taken together, we ought before we go any further to confider what will be the preffure which each of thefe indefinitely fmall Parts fuftains. Firft then we are to confider that every fmall particle of Water which is at reft is

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press'd upon equally on all sides by the other particles which surround it, otherwise it would yield to the stronger force till it were equally press'd every where, and as it is equally press'd on all sides so does it every way by reaction equally press what ever is contiguous to it according to all possible contrary directions, for should it press less than it were press'd it must necessarily yield to the force which is suppos'd greater then its own, and should it press more than it were press'd, its force would necessarily remove its weaker Antagonist. Therefore since all things are suppos'd to be at rest, we cannot any ways imagin this inequality of pressure to take place. Now it has been prov'd before that the pressure from above is equivalent to the weight of the incumbent Column of Water, therefore the pressure from any other part or according to any other direction is also equal to the weight of the same incumbent Column, and since action and reaction are equal, the particle it self must press according to all manner of directions, with the same force which is equivalent to the weight of the incumbent Column. 'Tis evident then that Fluids as they press according to all possible directions, so are the pressures equal according to all directions, if the Point of contact in which the pressure is made be at equal depths. This being allow'd we may proceed to what remains. Supposing then that *ACDB* (Fig. 6.) is a Cubical Vessel in which the Water reaches to the top, so that its upper Surface be represented by *AB*, let it be requir'd to determin the pressure which one of its sides *AC* does sustain from the included Water. This side *AC* though represented here by a Line, to avoid confusion in the Scheme, is suppos'd to be a Square. The measure of the pressure upon every Point of that Square (or as it is here represented of that Line *AC*) is the Altitude of the Water above that Point; thus the pressure upon *L* is measur'd by *AL*, the pressure upon *M* by *AM*, the pressure upon *N* by *AN*, and the pressure upon *C* by *AC*, and the same may be said for any other Points of the Line *AC*, therefore the pressure upon the whole Line or upon all the Points of it will be measur'd by the Sum of so many of those Altitudes *AL, AM, AN, AC*, as there are Points in the Line *AC*. Now that Sum may be thus estimated by drawing the Perpendicular *LO* equal to *AL* from the Point *L*, the Perpendicular *MP* equal to *MA* from the Point *M*, the Perpendicular *NQ* equal to *NA* from the Point *N*, and the Perpendicular *CD* equal to *CA* from the Point *C*. Now 'tis evident the Sum of *AL, AM, AN, AC* must be equal to the Sum of *LO, MP, NQ, CD*, and if from every intermediate Point between *A* and *L*, *L* and *M*, *M* and *N*, *N* and *C* Perpendiculars be conceiv'd to be drawn after the same method, the Sum of all those Perpendiculars will be the measure of the total pressure upon the Line *AC*. But the Sum of all those Perpendiculars is equal to the Area of the Triangle *ACD*, therefore the Area of the Triangle *ACD* is the measure of the pressure upon the Line *AC*. Now as the Line *AC* represents a Square so

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will the Triangle *ACD* reprefent a Prifm having the faid Triangle for its Bafe and the fide of the Square for its Altitude. The weight of that Prifm of Water is therefore equivalent to the preffure made againft the Square or fide of the Cube. That Prifm is equal to half the whole Cube as we learn from Euclid's Elements, therefore the preffure againft the Square is equivalent to half the weight of the whole Water contain'd in the Veffel. There are 4 fuch fides of a Cube befides the Top and Bottom, and each of thofe 4 fides for the fame reafon doth fuftain the fame preffure, therefore altogether do fuftain 4 times half the weight, that is twice the whole weight of the Water. And the Bottom by what has been prov'd above does it felf fuftain a preffure equal to the whole weight of the Water; therefore the Bottom and Sides together of a Cubical Veffel fill'd with Water do fuftain a preffure from the Water equal to thrice the weight of it. I have endeavour'd to make the thing as eafy as I believe the nature of it will permit. However fince that part of this deduction where I told You the Triangle *ACD* did at the fame time reprefent the Prifm when the Line *AC* reprefented the Square, might be perhaps a little obfcure I will endeavour to clear up this matter fomething further. Let then *ACFE* (Fig. 7.) reprefent the Square Side of the Veffel, and *CDGF* reprefent the Square Bottom of the fame. It was prov'd before that the preffure exercis'd upon the Line *AC* was meafur'd by the Triangle *ACD*, by the fame way of reafoning it may be prov'd that the preffure upon the Line *EF* is meafur'd by the Triangle *EFG*, and the preffure upon any other Line *HI* which is parallel to thefe two and fituated between them is meafur'd by its refpective Triangle *HIK*. If we imagin the Square *ACFE* to be made up of an Infinite Number of fuch intermediate Lines as *HI*, the preffure upon the whole Square will be made up of the fame Infinite Number of fuch equal Triangles as *HIK*, now the Sum of all thofe Triangles make up the Prifm *AEGDCF* and this Prifm is half the whole Cube, as in the former Scheme the Triangle *ACD* is half the Square *ACDB*. If the Plane *ACFE* inftead of being a Square were a rectangled Parallelogram having its Sides *AE* either longer or fhorter than *AC*, it would follow from the Principles that the preffure to which it is expos'd would be equivalent to the weight of a like Prifm of Water, having the Triangle *ACD* for its Bafis, and the Side *AE* for its Altitude.

I have been hitherto fpeaking of Planes which are either parallel or perpendicular to the Horizon, it will be no difficult matter to apply what has been faid of Perpendicular Planes to thofe which are oblique. Let *AC* (Fig. 8.) reprefent any fuch oblique Plane and let the upper Surface of the Water be *AB*. The meafure of the preffure upon the Point *L* is *LS* the Altitude of the Water above that Point, fo *TM* is the meafure of the preffure upon *M*, *VN* the meafure of the preffure upon *N*,

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and *XC* the meaſure of the preſſure upon *C.* Erect the Perpendiculars *LO, MP, NQ, CR* equal reſpectively to *LS, MT, NV, CX,* and imagin the like Conſtruction to be made for all the other Points of the Line *AC,* and the ſum of all thoſe Perpendiculars, that is the Triangle *ACR* will be the meaſure of the preſſure upon the whole Line *AC.* If this Line *AC* be ſuppos'd to repreſent a Parallelogram as before, then the Triangle *ACR* will as before become a Priſm, and the weight of that Priſm of Water, which we are taught by Euclid how to meaſure, will be the preſſure ſuſtain'd by the Parallelogram.

I have hitherto ſuppos'd that the Line *CA* or the Parallelogram repreſented by it coincides with the Surface of the Water at *A;* if that does not happen but the higheſt part of the Line or Parallelogram is at ſome diſtance from the Surface, a Computation of the preſſure will ſtill be eaſy enough. Suppoſe *MC* in the 8 Figure were the Line or Parallelogram propos'd, the Preſſure upon the Line *MC* will be meaſur'd by the Trapezium or four ſided Figure *MCRP,* and the preſſure upon the Parallelogram repreſented by that Line will be a Priſm, having that Trapezium for its baſe, and the other ſide of the Parallelogram which is ſuppos'd parallel to the Surface of the Water for its Altitude.

From what has been ſaid of theſe few particular Inſtances we may now underſtand, that the preſſure upon any Plane of what ever Figure and Situation is equivalent to the weight of a Solid of Water, which is form'd by erecting Perpendiculars upon every point of the Plane propos'd, equal to the reſpective diſtances of thoſe points from the upper Surface of the Water. For the Perpendiculars being the meaſure of the preſſure upon the Points from which they are erected, the Sum of thoſe Perpendiculars, or the Solid form'd by them, will be equal to the Sum of the preſſures upon the Points, or the total preſſure upon the whole Plane.

Or we may thus expreſs the ſame thing after another way, and ſo take in all curv'd Surfaces as well as Planes, that the preſſure upon any Surface is equal to the Sum of all the Products which are made by multiplying every indefinitely ſmall part of the Surface into its diſtance from the top of the Water. For the preſſure upon each of thoſe Parts is equal to a Column of Water having the Part for its Baſis and the diſtance from the top of the Water for its Altitude, and every one knows who has the leaſt skill in Geometry, that thoſe Columns are meaſur'd by multiplying their Baſes by their Altitudes, therefore the Sum of the Products of all thoſe Baſes or little Parts by their Altitudes or reſpective diſtances from the top of the Water will be equal to all the Columns upon every little Part, and therefore to a Body of Water whoſe weight will be equivalent to the total Preſſure upon the whole Surface. Now to find the Sum of all theſe Products or a Body of Water equal to that Sum, is a very difficult Problem in moſt Caſes. *Stevinus* in his Hydroſtaticks has attempted

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tempted it only in a few Instances, and those of plane Surfaces, and amongst plane Surfaces he meddles only with such which he calls regular, nevertheless he has gone the furthest in this matter of any Writer I have met with. To supply then this defect I will here lay down another Rule, which is not only universal, but also as easy and expeditious as can be desir'd. It is this, that the Pressure upon any Surface whatever, however it be situated is equal to the weight of a Body of Water whose magnitude is found by multiplying the Surface propos'd into the depth of its Centre of Gravity under Water. So the pressure upon any number of Surfaces of different Bodies, however differently situated, is equal to the weight of a Body of Water whose magnitude is found by multiplying the Sum of all those Surfaces into the depth of their common Centre of Gravity under Water. The Demonstration of this Rule may not perhaps be fully understood by those who are unacquainted with Staticks and the Nature of the Centre of Gravity, however I will here produce it, that those who can may understand it, and that others taking now for true what I shall assume as demonstrated by the Writers of Mechanicks may afterwards be fully satisfied of it, when they come to understand that Theorem it is grounded upon; which is, that if every indefinitely small Part of any Surface or number of Surfaces be multiplyed respectively into its Perpendicular distance from any propos'd Plane, the Sum of those Products will be equal to the Product of the whole Surface or number of Surfaces multiply'd into the Perpendicular distance of the Centre of Gravity of the single Surface, or of the common Centre of Gravity of the whole number of Surfaces from the same Plane. Now taking the upper Surface of Water for that Plane to which we refer the indefinitely small Parts of the Surface which is expos'd to the pressure we are concern'd with, since it has been already shewn that the pressure upon the whole is equivalent to the weight of a Body of Water which is equal in magnitude to the Sum of all the Products, made by multiplying every little Part by its distance from the upper Plane of the Water, and this Sum of Products is by the Statical Theorem I have been mentioning exactly equal to the Product of the whole Surface or number of Surfaces multiplyed into the distance of the Centre of Gravity from the upper Plane of the Water, it will follow that the same Product is the measure of a magnitude of Water whose weight is equivalent to the pressure requir'd. The same Rule may be demonstrated by several other methods, but I have pitch'd upon this as the fittest for my purpose.

Another thing which *Stevinus* proposes to himself is to determin the Centre of pressure upon any Plane. Before we can discourse any further about this we must declare what is meant by that Centre. It is then the point to which if the total pressure were apply'd, its effect upon the Plane would be the same as when it was distributed unequally over the whole

whole after the manner before describ'd; or we may say it is that Point in which the whole pressure may be conceiv'd to be united; or it is that point to which if a force were applyed, equal to the total pressure but with a contrary direction, it would exactly ballance or restrain the Effect of the pressure. Thus if *ABCD* (Fig. 9.) as before be a Vessel of Water, and the side *AC* be press'd upon with a force equivalent to twenty pounds of Water; this force we have seen is unequally distributed over *AC*, for the parts near *A* being at a lesser depth are less press'd upon than the parts near *C* which are at a greater depth, and therefore the efforts of all the particular pressures are united in some point *Z*, which is nearer to *C* than to *A*, and that point *Z* is what may be call'd the Centre of pressure: if to that point a force equivalent to twenty pound weight be apply'd it will affect the Plane *AC* in the same manner that it was affected before by the pressure of the Water distributed unequally over the whole. And if to the same point we apply the same force with a contrary direction to that of the pressure of the Water, the force and pressure will ballance each other, and by contrary endeavours destroy each others effects. Suppose at *Z* a Cord *ZPW* were fix'd, which passing over the Pulley *P*, has a weight *W* of twenty pounds annex'd to it, and that the part of the Cord *ZP* were perpendicular to *AC*, the effort of the weight *W* is equal and its direction contrary to that of the pressure of the Water. Now if *Z* be the Centre of pressure these two Powers will be in Equilibrio, & mutually defeat each others endeavours. It may be worth while to be acquainted with a Rule for finding that Centre in all Cases. We cannot have much help from *Stevinus* in this business, he undertakes only a few particulars and those which are the easiest, supposing that his Reader will apply the like method to other Circumstances, but they who shall endeavor to make such an Application will in most Cases find it more difficult than they might possibly expect. I have for that reason devis'd this general Rule which follows. That if any Plane which happens to be propos'd be produc'd till it intersect the upper Surface of the Water produc'd, if need be, and the Line which is the common Section of the two Planes be made an Axis of Suspension, the Centre of Oscillation or Percussion of the Plane as it is suppos'd to revolve about that Axis will be the Centre of pressure requir'd. Thus if *AC* (Fig. 10.) represents the Plane propos'd let it be produc'd till it cut the Plane *HG* in *D*, now if *D* be made the Axis of Suspension of the Plane *AC*, the Centre of percussion of the Plane *AC* revolving about *D* will be also the Centre of pressure upon the same Plane. For if the percussive forces of every point of *AC* be as the pressures exercis'd upon those points, then the Centre of percussion must needs be the same with the Centre of pressure, and that the force of percussion is every where as the pressure of the Water may thus be prov'd. The percussive force of any point suppose *B* is as the velocity of that point, and

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the velocity is as the distance *BD* of the point from the Axis of Motion; so the percussive force of *A* is as *AD*, of *C* as *CD*; since then the percussive forces of *A, B, C* are as the Lines *DA, DB, DC* and those Lines are as the Lines *EA, FB, GC* perpendicular to the Surface of the Water, and these last Lines are as the pressures upon *A, B,* and *C;* it follows that the percussive forces taking the intersection *D* for the Axis of suspension or motion are respectively as the pressures upon the same points : therefore the Centre of Percussion or Oscillation is the same with the Centre of pressure. The Geometers of the last Age have prosecuted the Problem of finding the Centre of Oscillation very diligently, being excited thereto chiefly by the noble Invention of Pendulum Clocks: the Rules they have laid down for that purpose are easy enough, and the Applications they have actually made of those Rules are not a few. Having therefore shewn how the Centre of Oscillation may be made use of for determining the Centre of pressure, I presume I have by this time sufficiently clear'd up what I propos'd. For further illustration I will add a couple of Examples.

Let it be requir'd to find the pressure which a Diver sustains when the Centre of Gravity of the Surface of his Body is 32 feet under Water. The Surface of a middle siz'd humane Body is about 10 square feet. Multiply then 32 the depth of the Centre under Water by 10 the Surface of the Body, and the product or 32 times 10 solid feet will be a magnitude of Water whose weight is equivalent to the pressure which the Diver sustains by the Rule before laid down. A Cubick foot of Water has been found by Experiment to weigh 1000 Averdupois Ounces, therefore 32 times 10 feet or 16 times 20 feet of Water will weigh 16 times 20000 Averdupois Ounces or 20000 Averdupois Pounds. This therefore is the pressure of the Water to which a Diver at 32 feet depth is expos'd. Again in (Fig. 11.) let the right angled Parallelogram *ABCD* be a Wall, Dam, or Pen of Timber perpendicular to the Horizon made to keep in a Pond of Water, whose upper Surface reaches to *AB*: let *AB* be 20 feet, and *AC* 12. Let *K* be the Centre of Gravity of the Plane, the depth of that Centre *K* will be equal to half *GH* or half *AC*, that is 6 feet. The Area of the Plane is found by multiplying *AC* by *AB* or 12 by 20, it is therefore 240 square feet: multiply according to the Rule the Area 240 by *GK* which is 6 and the product will be 1440 Cubick feet of Water, which weighs so many thousand Ounces that is 90000 pounds, and that is the pressure which the Dam *ABCD* sustains. To find the Centre of that pressure we must make the Line *AB* which is the common Section of the Dam and the upper Surface of the Water, the Axis of suspension of the Plane *ABCD*; now it appears by the discovery of *Hugens, Wallis* and other Geometers that *Z* the Centre of Oscillation of this Plane so suspended will be in the Line *GH*

*GH* which bisects this Plane and is parallel to *AC* or *BD*, and the Line *GZ* will be two thirds of *GH* or *AC* that is 8 feet and the same point *Z* so determin'd is, as was prov'd before, the Centre of pressure requir'd.

## An Enquiry into the Limits and State of the Atmosphere.

HAVING Yesterday made it appear from reason that the Spring or Elastick Power of the Air is as the force which compresses it, and having this Day as far as the unavoidable irregularity of Tubes would permit us, shewn by several Experiments that the Density is also as the same force, the Space it possesses being always reciprocally as that force; We are now furnish'd with sufficient *data* to make our Enquiries concerning the Limits of the Atmosphere, and to determine its State as to Rarity at different Elevations from the Earth's Surface. If the Air were of the same consistence as to its Rarity or Density at all Altitudes it would be no difficult thing to set bounds to it. We collected from the Experiment which was Yesterday made at the top and bottom of the Observatory, that the Specifick Gravity of Water is about 850 times greater than the Specifick Gravity of Air, (which thing will hereafter be further examin'd by an Experiment particularly fitted for that purpose) and in the foregoing Week we found by the Hydrostatical Balance, that Quicksilver is about 14 times heavier than Water; it follows then of consequence that Quicksilver is 14 times 850 degrees heavier than Air, that is, 11900 times heavier. We have seen by the Torricellian Experiment that a Column of Quicksilver of $29\frac{1}{2}$ Inches is usually a counterpoise to a Column of Air having the same basis and reaching to the top of the Atmosphere; if therefore the Air be every where of the same Density as it is here below, its Altitude ought as many times to exceed the height of $29\frac{1}{2}$ Inches (which is the height of an Æquiponderant Column of Quicksilver) as its Specifick Gravity falls short of the Specifick Gravity of Quicksilver; that is, the Atmosphere ought ( upon the supposition of an every where uniform Density) to be 11900 times $29\frac{1}{2}$ Inches or somewhat above $5\frac{1}{2}$ Miles high. But it may be easily prov'd that this supposition does in no wise take place. For since every Region of the Air is comprest by that part of the Atmosphere which is superior to it, and since the higher Parts have a lesser weight incumbent upon them than the lower, and since the Density of the Air is every where as the force which compresses it, it will follow of necessity that there is still a greater Rarity of the Air as it is further distant from the Surface of the Earth. How far the Air may possibly admit of Rarefaction and Condensation has not yet that I know of been determin'd by any one. Mr. *Boyl* has observ'd that it may be so dilated as to become 10000 times rarer than it is in its natural State.

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Dr. *Halley* says that he himself has seen the Air compress'd so as to be 60 times denser than it is as we commonly breath it; and Monsieur *Papin* relates that he was a Witness that Monsieur *Hugens* did once in a Glass Vessel compress the Air to the same degree before the Glass was broken, yet never could any Experimenter determin how much further the Air might possibly be rarefied or condens'd. However it's certain that there are in Nature some Limits which cannot be exceeded. No condensation can reach so far as to cause a penetration of Parts, and if the Rarefaction of the Air be still greater, as its distance from the Surface of the Earth increaseth, its Spring will at length be so weakn'd that the force, with which every Particle of it endeavours to tend upwards from the Particles which are next below it, will be weaker than the force of its own gravity which endeavours constantly to detain it. The Rarefaction of the Air must therefore be bounded of necessity when these two opposite forces come to ballance each other. Though this be certainly true that the Air can't possibly expand it self beyond a certain measure upon account of its Gravity, yet since Men have not hitherto been able to set any bounds to its utmost expansion, it is equally certain, that we cannot possibly define the Limits of the Atmosphere. For as the Air may be more and more rarefied, so will the same Quantity of it (which equals the weight of about 30 Inches of Quicksilver) be contain'd in a greater Space, and thereby those Limits be so much the wider. Notwithstanding this seeming difficulty we may still collect how much the Air is rarified at any propos'd Altitude from the Surface of the Earth after the following manner. Let *XAαPX* (Fig. 14.) represent a Vessel reaching from the Surface of the Earth *Aα* to the top of the Atmosphere *X*. Let us imagin the side *AX* divided into Inches *AB, BC, CD* &c. and let the Lines *BK, CL, DM, EN* &c be drawn parallel to *Aα*. 'Tis evident that the Air contain'd between *BK* and *CL* is rarer than the Air contain'd between *Aα* and *BK*; the former having a lesser Column of Air *XCLX* incumbent upon it, than the Column *XBKX* which presses upon the latter. Upon the same account the Air between *CL* and *DM* is rarer than that between *BK* and *CL*, and that between *DM* and *EN* rarer than that between *CL* and *DM*. And thus is every superior Inch of Air rarer than that below it. Let us now suppose, that every Inch of Air is in all parts of it of an equal Density, or that the Air *AK* is every where uniform, but denser than the Air *BL*; which is also suppos'd to be every where uniform, but denser than *CM*; and that to be uniform it self, but denser than *DN*, and so onwards. Again let us suppose that the Air *BL* is reduc'd to a lesser Space *BQ*, so as to become equally dense with the Air *AK*, which is done by making the Space *BQ* lesser than *BL*, in the same proportion that the Air *BL* is less dense than the Air *AK*; after the same manner let the Air *CM* be reduc'd to the Space *CR*, and the Air

*DN*

*DN* to the Space *DS* and fo onwards, that thus every Inch of Air may be reduc'd to the fame confiftence with the Air *AK*; now it is evident from this conftruction that the Spaces *AK*, *BQ*, *CR*, *DS*, &c will every where be as the Denfities refpectively of the feveral Inches of Air *AK*, *BL*, *CM*, *DN*; and it is alfo evident that the Quantity or weight of the Air which reaches from any one of thofe Spaces up to the extremity of the Atfphere will every where be as the Sum of all the Spaces which are fituated above the Space propos'd. Thus the Quantity or the weight of Air above the Space *AK* will be as the Sum of the Spaces *BQ*, *CR*, *DS*, *ET*, *FV* &c, and the Quantity or weight of Air above the Space *CR* will be as the Sum of the Spaces *DS*, *ET*, *FV* &c. For the Air being every where reduc'd to the fame confiftence, the Quantity or weight of it will be as the Space it poffeffes. Thefe things being laid down I may now without much difficulty proceed to eftablifh the Conclufion I aim at, which is this, that if any number of diftances from the Surface of the Earth be taken in an Arithmetical Progreffion, the Denfities of the Air at thofe Diftances will be in a Geometrical Progreffion. For fince by the Experiments which have this Day been made, it appears that the Denfity of the Air is always as the force which compreffes it, we muft conclude that the Denfity of the Air at any Diftance from the Surface of the Earth is as the Quantity or weight of that part of the Atmofphere which is above it. Therefore in our Scheme the Denfities of the Air between *Aα* and *BK*, *BK* and *CL*, *CL* and *DM* &c. are to each other refpectively as the Quantities of Air Above *Aα*, *BK*, *CL* &c. up to the extremity of the Atmofphere. But we faw before that thofe Denfities were as the Spaces *AK*, *BQ*, *CR* &c. refpectively, and thofe Quantities of Air reaching to the extremity of the Atmofphere were as the Spaces *XBβQRSTVX*, *XCγRSTVX*, *XDδSTVX* refpectively, it follows then that the Spaces *AK*, *BQ*, *CR* are to each other refpectively as the Spaces *XBβQRSTVX*, *XCγRSTVX*, *XDδSTVX*. Now the former Spaces *AK*, *BQ*, *CR* are the differences of the latter, and it is well known to thofe who underftand any thing of the nature of Proportions, when any fet of Quantities are to each other refpectively as their differences that then as well the Quantities themfelves as their differences, are in a Geometrical Progreffion. The Spaces *AK*, *BQ*, *CR*, are therefore in a Geometrical Progreffion, as the Diftances *AB*, *AC*, *AD* are in an Arithmetical Progreffion. And as the Denfities of the Air belonging to thefe three firft Inches are in a Geometrical Progreffion, fo do the Denfities of the Air belonging to every one of the other Inches, which are fuppos'd to be continued up to the extremity of the Atmofphere, decreafe in the fame Geometrical Progreffion, as any one without difficulty may collect by the fame way of reafoning. I have hitherto fuppofed for eafe of conception that the Air is of the fame Denfity in every part of each Inch of Alti-

tude.

tude, nevertheless it is certain that every the least variation of Altitude causes a variation of Density in the Air. The Conclusion however will not hereby be disturb'd: for if instead of dividing the Altitude of the Atmosphere into Inches as before, we conceive it now to be divided into its most indefinitely minute parts, applying to these what we have said above concerning the Inches, we shall at length deduce the same Geometrical Progression of Densities answering to a like Arithmetical Progression of Altitudes. Now because the Rarity of any Body is reciprocally as its Density, we may also conclude that as the Distances from the Surface of the Earth do increase in an Arithmetical Progression, so do the different degrees of Rarity of the Air increase in a Geometrical Progression. This Property of the Air was first that I know of observ'd by Dr. *Halley*, but because his Demonstration cannot be understood by those who are unacquainted with the nature of the Hyperbolick Line, and Dr. *Gregory* in his Demonstration of the same thing, which may be seen in the fifth Book of his Astronomy, supposes his Reader to be furnish'd with so much Geometry as not to be ignorant of the properties of the Logarithmick Line, I have endeavour'd to make the thing intelligible by a Method which may be easy even to those who have never medled with Curvilinear Figures. Let us see now what help we have from this property to determin how much the Air is really rarefied at any propos'd Elevation from the Surface of the Earth. Since the Elevations are the Terms of an Arithmetical Progression as the Rarities are the Terms of a Geometrical, it follows, that the Elevation is every where proportionable to the Logarithm of the Rarity. If then by Experiment we can possibly find the Rarity of the Air at any one Elevation, we may by the Rule of Proportion find what is the Rarity at any other propos'd Elevation: by saying, as the Elevation at which the Experiment was made, is to the Elevation propos'd, so is the Logarithm of the Air's Rarity which was observ'd at the Elevation where the Experiment was made, to the Logarithm of the Air's Rarity at the Elevation propos'd. Thus I collected from the celebrated French Experiment of the *Puy de Domme*, which I Yesterday gave You an Account of, that at the Altitude of 7 Miles the Air is somewhat above 4 times rarer than at the Surface of the Earth. By the same method I collected from the Experiment of Mr. *Caswell* made upon *Snowden Hill*, that at the same Altitude of 7 Miles the Air is not altogether so much as 4 times rarer than at the Surface, the difference on both sides was inconsiderable. We may take a mean therefore and say in a round number, that at the Altitude of 7 Miles the Air is about 4 times rarer than at the Surface of the Earth. Sr. *Isaac Newton* in his late Additions to his Opticks makes use of this very same proportion, what grounds he went upon is difficult to guess, however I am satisfied of the Conclusion from my own Computation. Now from what has been already prov'd that the

Rarity

Rarity of the Air is augmented in a Geometrical as the Altitude is augmented in an Arithmetical Progreſſion, it follows that every ſeven Miles added to the Altitude does always require a rarity of the Air ſtill 4 times greater. Therefore at the Altitude of 14 Miles the Air is 16 times rarer than at the Surface, at the Altitude of 21 Miles it is 64 times rarer, at the Altitude of 28 Miles 256 times, at 35 Miles 1024 times, at 70 Miles about a Million of times, at 140 Miles a Million of Million of times, at 210 Miles a Million of Million of Millions of times, if the Air can poſſibly expand it ſelf to ſo large Dimenſions. Hence we may eaſily gather that the Air at the Altitude of 500 Miles (if the Atmoſphere can reach ſo far) muſt neceſſarily be there ſo much rarified that if a Globe of the Air we breath in of an Inch Diameter were as much dilated it would poſſeſs a larger Space than the whole Sphere of Saturn. The Semidiameter of the Earth is nearly 4000 Miles which is 8 times 5 hundred Miles: with good reaſon then might that excellent Philoſopher I have lately been mentioning tell us in his *Principia*, that the Air at the Altitude of a Semidiameter of the Earth is at leaſt ſo wonderfully rarified as I have deſcrib'd it to be at an Altitude 8 times leſs.

It appears from the Obſervations of Aſtronomers of the Duration of Twilight and of the magnitude of the Terreſtial Shadow in Lunar Eclipſes, that the Effect of the Atmoſphere to reflect and intercept the light of the Sun is ſenſible even to the Altitude of between 40 and 50 Miles, ſo far then may we be certain that the Atmoſphere reaches, and at that Altitude we may collect from what has been already ſaid that the Air is about 10000 times rarer than at the Surface of the Earth. How much farther than this Altitude of between 40 and 50 Miles the Atmoſphere may be extended I muſt confeſs I am altogether ignorant, there bring no *data* that I know of from which a greater Altitude may be indubitably concluded. There has indeed been often ſeen in the Atmoſphere ſome very Luminous parts even near the Zenith about Midnight, but I dare not conclude any thing from ſuch appearances; if I ſhould aſſert as ſome have done that theſe luminous parts are nothing elſe but ſome Terreſtial Exhalations floating in the Air at a prodigious Altitude, and thereby reflecting the Light of the Sun which they are expos'd to at that great height to our eyes, it will be next to impoſſible to give any tolerable account how thoſe Exhalations can be denſe enough to reflect ſo copious a Light at that vaſt diſtance, and at the ſame time be ſupported by a Medium, I may ſay, almoſt infinitely rarer than the Air we breath in. It ſeems more probable that theſe extraordinary Lights proceed from ſome ſelf ſhining Subſtance or Aerial *Phoſphorus*. A ſurprizing appearance of this kind was ſeen at *Cambridge* about 10 of the Clock at Night and at other very diſtant Places on the 20*th* of *March* in the Year 1706. It was a Semicircle of Light of about two thirds of the ordinary breadth of the milky

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way but much brighter. The top of it pafs'd very near our Zenith inclining about 4 or 5 Degrees to the North, it crofs'd the Horizon at a very fmall diftance from the Weft towards the South, and again about as far from the Eaft towards the North. It was moft vivid and beft defin'd about the Weftern Horizon and moft faint about the Zenith, where it firft began to difappear : there was at the fame time an *aurora Borealis.* A Friend of mine faw the fame appearance in *Lincolnfhire* at the diftance of about 70 Miles north of *Cambridge* : the Semicircle feem'd to Him to lye in the Plane of the Æquator. From thefe two obfervations compar'd together it is eafy to collect that the matter from which that Light proceeded was elevated above the Earth's Surface between 40 and 50 Miles. Having now finifh'd what I defign'd to reprefent concerning the Limits and different degrees of rarity of the Atmofphere at different Altitudes I might here conclude. But becaufe it may poffibly be expected that I fhould add fomething in this place concerning the Caufe of the Airs Elafticity upon which thefe deductions were grounded, it may not be amifs to declare here that of all the feveral Hypothefes which I have hitherto been fuggefting for this purpofe, that of Sir *Ifaac Newton* feems to me to be the moft probable. He has demonftrated in the fecond Book of his *Principia* that if the particles of the Air be of fuch a Nature as to recede from each other with centrifugal forces reciprocally proportionable to their diftances, they will compofe an Elaftical Fluid whofe Denfity will always be as the force which compreffes it; and any one who reads the late Additions to his Opticks will perceive that that Hypothefis is not advanc'd without reafon.

*An account of the feveral fucceffive Degrees in which the Air is expanded and compreffed by the Air-pump and Condenfer. Wherein the firft and fecond Tables are explained.*

AT our laft meeting we took a particular view of the feveral parts of which our Engines confift. I fhall therefore fuppofe You to be fufficiently acquainted with the Fabrick and contrivance of them, and to underftand in general the manner of their Operations. I fay in general, becaufe there are fome particulars which yet remain at this time to be difcours'd of, which may alfo very well deferve your confideration and will be of good Ufe in order to frame juft and true apprehenfions of the Experiments which will hereafter be made. I fhall begin with the Air-pump, and reprefent to You by what degrees the Air contain'd in the Receiver is exhaufted.

It may perhaps upon the firft view feem not improbable that an equal evacuation is made at each ftroke of the Pump and confequently that the

Receiver

Receiver may after a certain number of Strokes be perfectly exhausted; for it muſt be allow'd, if an equal quantity of Air is taken away at every ſtroke, that the Receiver will in time be perfectly exhauſted, how ſmall ſoever thoſe equal quantities which are continually taken away, may be ſuppoſed to be. Thus if the Air which goes out of the Receiver at each turn of the Pump be but the Hundredth part of what was at firſt includ-ed in the Receiver; 'tis certain that a total evacuation will be made after an Hundred turns. That things are thus may at firſt view I ſay ſeem not improbable. But if we conſider the matter more nearly we ſhall find it to be far otherwiſe.

What I ſhall endeavour to make out to you is this; that the quanti-ties exhauſted at every ſtroke are not equal but are perpetually diminiſh'd and grow leſſer always ſo long as you continue to work the Pump: that no Receiver can ever be perfectly and intirely evacuated, how long time ſoever you imploy for that purpoſe, notwithſtanding that the Engin be abſolutely free from all defects and in the greateſt perfection which can be imagin'd. It may appear to be a Paradox, that a certain quantity of the Air in the Receiver ſhould be removed at every turn of the Pump and yet that the whole can never be taken away; but I hope I ſhall eaſily ſatisfy you that it is not a Miſtake. Laſtly, that I may not ſeem too much to depreciate the Value of our Engin, I have this further to ſay for it: that though it be impoſſible by it's means to procure a perfect Vacuum, yet you may approach as near to it as you pleaſe. By a perfect Vacuum here I mean in reſpect of Air only, not an abſolute Vacuity in reſpect of every thing which is Material: for not to mention what other ſubtile Bodies may poſſibly be lodg'd in our emptied Receivers, 'tis matter of fact that the Rays of Light are not excluded from thence.

In Order to make out theſe Aſſertions, I ſhall in the firſt place lay down this Rule. That the quantity of Air which is drawn from the Receiver at each ſtroke of the Pump, bears the ſame proportion to the quantity of Air in the Receiver immediately before that ſtroke, as the capacity of the Barrel into which the Air paſſes from the Receiver, does to the capacity of the ſame Barrel and the capacity of the Receiver taken together. You may remember that in each Barrel there are two Valves, whereof the lower is placed at the bottom of the Barrel and the upper is fix'd upon the Embolus or Sucker. Now the hollow Space which lies betwixt theſe Valves, when the Embolus is rais'd as high as it can go, is what I call the capacity of the Barrel: for the other part of the cavity of the Barrel, which is above the Embolus and the upper Valve, is of no uſe in evacuating the Receiver, and therefore ought not here to be conſidered. Upon a like account, by the capacity of the Receiver, I mean, not only the Space immediately contained under the Receiver, but alſo all thoſe other hollow Spaces which communicate with it, as far as to

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the lower Valves: such you may remember are the cavity of the Pipe which conveys the Air to the Barrels, and the cavity in the upper part of the Gage above the Quickſilver. Theſe additional Spaces are very ſmall and inconſiderable; yet if we would be exact, they alſo muſt be taken into the account and look'd upon as parts of the Receiver. Now to underſtand the truth of this Rule, we muſt obſerve that as the Embolus is moved upwards from the bottom of the Barrel, it would leave a void Space behind it, but this effect is prevented by the ruſhing in of Air from the Receiver. The Air you know, by it's Elaſticity, is always endeavouring to expand it ſelf into larger Dimenſions, and it is by this endeavour that it opens the lower Valve and paſſes into the hollow part of the Barrel as the Embolus gives way to it, and this it will continue to do, till it comes to have the ſame Denſity in the Barrel as in the Receiver: for ſhould its Denſity in the Barrel be leſs than in the Receiver, its Elaſtick force which is proportionable to its Denſity would be leſs alſo, and therefore it muſt ſtill give way to the Air in the Receiver till at length the Denſities become the ſame. The Air then which immediately before this ſtroke of the Pump, (by which the Sucker is raiſed) was contained in the Receiver only, is now uniformly diffuſed into the Receiver and the Barrel; whence it appears that the quantity of Air in the Barrel is to the quantity of Air in the Barrel and Receiver together as the capacity of the Barrel is to the capacity of the Barrel and Receiver together. But the Air in the Barrel is that which is excluded from the Receiver by this ſtroke of the Pump, and the Air in the Barrel and Receiver together is what was in the Receiver immediately before the ſtroke. Therefore the truth of the Rule is very evident: that the quantity of Air which is drawn from the Receiver at each ſtroke of the Pump bears the ſame proportion to the quantity of Air in the Receiver immediately before that ſtroke, as the capacity of the Barrel into which the Air paſſes from the Receiver, does to the capacity of the ſame Barrel and the capacity of the Receiver taken together. To illuſtrate this further by an Example; let us ſuppoſe the capacity of the Receiver to be twice as great as the capacity of the Barrel; then will the capacity of the Barrel be to the capacity of the Barrel and Receiver together as 1 to 3, and the quantity of Air exhauſted at each turn of the Pump is to the quantity of Air which was in the Receiver immediately before that turn, in the ſame proportion. So that by the firſt ſtroke of the Pump, a third part of the Air in the Receiver is taken away; by the ſecond ſtroke a third part of the remaining Air is taken away, by the third ſtroke a third part of the next remainder is exhauſted, by the fourth a third part of the next, and ſo on continually; the quantity of Air evacuated at each ſtroke diminiſhing in the ſame proportion with the quantity of Air remaining in the Receiver immediately before that ſtroke: for 'tis very evident that the third

part,

part, or any other determinate part, of any Quantity muſt needs be diminiſhed in the ſame proportion with the whole Quantity it ſelf. And this may ſuffice for the proof of what I aſſerted in the firſt place _viz._ That the Quantities exhauſted at every ſtroke are not equal but are perpetually diminiſhed.

I ſhall now proceed to ſhew, that the Air remaining in the Receiver after every ſtroke is diminiſhed in a Geometrical Progreſſion. It has been prov'd that the Air remaining in the Receiver after each ſtroke of the Pump is to the Air which was in the Receiver immediately before that ſtroke, as the capacity of the Receiver is to the capacity of the Barrel and Receiver taken together, or in other words, that the quantity of Air in the Receiver, by each ſtroke of the Pump, is diminiſh'd in the proportion of the capacity of the Receiver to the capacity of the Barrel and Receiver taken together. Each remainder is therefore evermore leſs than the preceding remainder in the ſame given _Ratio._ That is to ſay theſe remainders are in a Geometrical Progreſſion continually decreaſing. Let us return again to our former Example which may afford a ſomewhat different Light into this matter. The Quantity exhauſted at the firſt turn, you remember, was a third part of the Air in the Receiver, and therefore the remainder will be two thirds of the ſame, and for the like reaſon the remainder after the ſecond turn will be two thirds of the foregoing remainder, and ſo on continually, the decreaſe being always made in the ſame proportion of 2 to 3 ; conſequently the decreaſing Quantities themſelves are in a Geometrical Progreſſion. It was before proved that the Quantities exhauſted at every turn did decreaſe in the ſame proportion with theſe remainders; therefore the Quantities exhauſted at every turn are alſo in a Geometrical Progreſſion. Let it then be remembred, that the Evacuations and the Remainders do both of them decreaſe in the ſame Geometrical Progreſſion. If the Remainders do decreaſe in a Geometrical Progreſſion, 'tis certain you may, by continuing the Agitations of the Pump, render them as ſmall as you pleaſe, that is to ſay, you may approach as near as you pleaſe to a perfect Vacuum. But notwithſtanding this, you can never entirely take away the Remainder. If it be ſaid that you may, I prove the contrary thus. Before the laſt turn of the Pump, which is ſaid wholly to take away the Remainder, it muſt be confeſs'd there was a Remainder: this Remainder, by that laſt turn of the Pump, will only be diminiſhed in a certain proportion as has been before proved: therefore it was falſly ſaid to be totally taken away.

It may not be improper in this place to ſay ſomething concerning the Gradual Aſcent of the Quickſilver in the Gage, upon which we have made ſome Experiments. You have obſerv'd that as we continue to Pump, the Quickſilver continues to aſcend, approaching always more and more to the Standard Altitude in the Weather-Glaſs, which you know is about

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bout 29½ Inches, being a little under or over according to the variety of Seasons. What I shall now endeavour to make out to you is this: that the defect of the height of the Quickfilver in the Gage from the standard Altitude, is always proportionable to the Quantity of Air, which remains in the Receiver: that the Altitude it self of the Quickfilver in the Gage, is proportionable to the Quantity of Air which has been exhausted from the Receiver: that the Ascent of the Quickfilver upon every turn of the Pump, is proportionable to the Quantity evacuated by each turn.

In order to understand these Assertions, you are to consider, that the whole pressure of the Atmosphere upon the Cistern of the Gage, is equivalent to and may be balanc'd by a Column of Quickfilver of the Standard Altitude. Therefore when in the Gage, the Quickfilver has not yet arrived to the Standard Altitude, 'tis certain the defect of Quickfilver is supplyed by some other equal force, and that force is the Elastick Power of the Air yet remaining in the Receiver, which communicating (as you remember) with the upper part of the Gage, hinders the Quickfilver from ascending, as it would otherwise do, to the Standard Altitude. The Elasticity of the Air in the Receiver is then equivalent to the weight of the deficient Quickfilver: but the weight of that deficient Quickfilver is proportionable to the Space it should possess, or to the defect of the height of the Quickfilver in the Gage from the Standard height: therefore the Elasticity of the remaining Air is also proportionable to the same defect. And since it was formerly proved, that the Density of any portion of Air is always proportionable to its Elasticity, and the Quantity in this Case is proportionable to the Density; it follows, that the Quantity of Air remaining in the Receiver, is proportionable to the defect of the Quickfilver in the Gage from its Standard Altitude, which was the first thing to be proved. Hence it follows, that the Quantity of Air which was at first in the Receiver before you began to Pump, is proportionable to the whole Standard Altitude, and consequently the difference of this Air which was at first in the Receiver and that which remains after any certain Number of turns, that is, the Quantity of Air exhausted, is proportionable to the difference of the Standard Altitude and the beforemention'd defect, that is, to the Altitude of the Quickfilver in the Gage after that Number of turns; which was the second thing to be proved. And from hence it follows that the Quanty of Air exhausted at every turn of the Pump, is proportionable to the Ascent of the Quickfilver upon each turn, which was the last thing to be made out. And these Conclusions do very well agree with the Experiments, which shew'd us the quantity of Air that was exhausted by the Quantity of Water which afterwards supply'd the vacant Place of that Air in our Receiver. Let it then be remembred, that the Quantity exhausted at each turn is proportionable

the Afcent of the Quickſilver upon that turn: that the whole Quantity exhauſted from the time you began to pump is proportionable to the whole Altitude of the Quickſilver: that the quantity remaining in the Receiver is proportionable to the defeć of that Altitude from the Standard; to come now to the application of the other Experiments which we made this Day: We found you remember, that the ſeveral Afcents of the Quickſilver in the Gage, upon every turn of the Pump, were diminiſhed in a Geometrical Progreſſion, and it has juſt now been proved that the quantities of Air exhauſted at each turn are proportionable to thoſe Afcents. Therefore we may ſafely conclude from Experiment alſo, what we before collećted by a train of reaſoning: that the Quantities of Air exhauſted at every turn of the Pump are diminiſhed continually in a Geometrical Progreſſion. Furthermore, ſince thoſe Afcents are the differences of the defećts from the Standard Altitude, upon every ſucceſſive turn of the Pump: it follows, that the defećts alſo are in the ſame decreaſing Geometrical Progreſſion. For 'tis a general Theorem, that all Quantities, whoſe differences are in a Geometrical Progreſſion, (ſo long as the Quantities continue to have any magnitude,) are themſelves alſo in the ſame Geometrical Progreſſion. The defećts being then in a decreaſing Geometrical Progreſſion, and the Quantities of Air remaining in the Receiver being proportionable (as was lately proved) to the defećts: it follows from the ſame Experiments, that the Quantities of Air which remain in the Receiver after every turn of the Pump, do decreaſe in a Geometrical Progreſſion: which was the other thing concluded alſo by a train of reaſoning.

Before I diſmiſs the conſideration of the Air-Pump, it remains that I add ſomething concerning the uſe of the two Tables, which I have put into your hands. They are deſigned to ſhew the number of turns of the Pump, which are requiſite to rarefie, in any given proportion, the Air contain'd under any Receiver. The firſt Table in particular is fitted for Receivers whoſe capacity is the ſame with the capacity of the Barrel, and the Numbers of the firſt Column of it expreſs the degrees of Rarefaćtion as thoſe over againſt them in the ſecond Column expreſs the Number of Turns, with their Decimal Parts, which are requiſite to produce thoſe degrees of Rarefaćtion. Thus for Example, if it were required to rarefie the Air, under ſuch a Receiver, an hundred times above its natural Rarity: I ſeek for the Number 100 in the firſt Column, and over againſt it in the ſecond I find the Number 6,644 by which I underſtand that the Air will be rarified an Hundred times by 6 turns of the Pump and 644 Thouſand parts of a turn. So if it were deſired to rarefie the Air, under the ſame or an equal Receiver, 10 Thouſand times more than in its natural ſtate: I perceive there will be 13 turns and 288 Thouſandth parts of a turn requiſite for that purpoſe. The Receivers which we ſhall have occaſion to make uſe of in our Experiments are generally much bigger

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than the capacity of each Barrel of the Pump, and by being bigger, will require a greater number of turns than those set down in the second Column, to rarifie the Air in the degrees which are express'd in the first Column. It may perhaps at the first view, seem not unreasonable to think that the number of turns requisite to rarefie the Air in any certain Degree, should exceed the Numbers of the second Column in the same proportion by which the capacity of the Receiver exceeds the capacity of the Barrel. But if the Matter be examined more closely, it will be found: that the Number of turns do not increase in so great a proportion as the capacity of the Receiver does. What that proportion is, by which the Number of turns is truly increased, as the capacity of the Receiver becomes bigger, may be seen by the second Table. Whose first Column expresses the proportion of the Receiver to the Barrel as the second does the proportion of the true Number of turns to those set down in the first Table. The use of it will be more clearly understood by an Example or two. Let us suppose the capacity of the Receiver to be 10 times greater than the capacity of the Barrel and that we would find how many turns are requisite to rarefie the Air under such a Receiver 100 times more than it is naturally rarefied. By the first Table we find (as was said above) that if the Receiver were equal to the Barrel the Number of turns would be 6,644. But the Receiver is 10 times greater. Find therefore the Number 10 in the first Column of the second Table, and over against it you will see the Number 7,273 in the second Column of the same Table, by which you perceive that as the Receiver is increased in a Decuple proportion, the Number of turns are increased not so much, but only in a somewhat more than Septuple proportion. Therefore the true Number of turns will be found by multiplying the Number 6,644 by the Number 7,273 and will consequently be 48, 322. So if it were desired to find the Number of turns of the Pump, which must be made, to rarefie the Air 10 Thousand times above its natural State, in a Receiver which is 50 times bigger than the capacity of the Barrel: over against 10000 in the first Table I find 13,288 and over against 50, in the second Table, I find 35,003 which multiplyed together make 465,12; This therefore is the Number of turns requisite for the purpose. You need not be solicitous about the Fractions which are above any certain whole Number of turns. They do not mean, that the Handle of the Pump is to be moved justly such a part of a turn as they seem to denote; for strictly speaking it need not be moved altogether, so much. But the difference is inconsiderable, and it would be a loss of time to insist more particularly about it. It was necessary to set down the Fractions in the Tables that no whole Number of turns might be lost in the Product, when you come to multiply 'em together; but when you have found the Product, the Fractions belonging to it need not be considered.

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In making thefe Tables, that they might not be too large, I have as you fee omitted feveral intermediate Numbers. However, they are fufficient for the purpofe for which I defign'd 'em; which was to give you clearer Notions of the Operation of our Engin. I fhould here explain to you the grounds upon which they were computed; but I fear the difficulty of the Subject would not permit me to be generally underftood. I fhall therefore omit the doing of it, and only obferve to you of the firft Table, that if you take any Numbers in the firft Column which are in a Geometrical Progreffion, the correfpondent Numbers of the fecond Column will be in Arithmetical Progreffion. It may alfo be obferved of the fecond Table that the difproportion of the correfpondent Numbers does continually increafe from the beginning to the end, how far fo ever it be continued, but yet does never exceed the difproportion of 13 to 9.

'Tis time now that we proceed to the Condenfer. This Inftrument will not require much to be faid concerning it. When I affert that equal Quantities of Air, namely, as much as the Barrel can naturally contain, are intruded into the Receiver at each ftroke of the Forcer: the thing is fo very obvious that I believe I need not go about to prove it. For you cannot but eafily underftand, that as the Embolus or Forcer is drawn upwards from the bottom of the Barrel, there is a vacuity left behind it, till fuch time as it comes to get above the little Hole which is made in the fide of the Barrel towards the top of it. For then the external Air is permitted to pafs freely through that Hole into the aforefaid void Space, and confequently the Barrel will then have as much Air in it as it can naturally contain. And as the Forcer is moved downwards this Air is comprefs'd, and by compreffion is more and more condens'd till at length the force of its Elafticity becomes greater than the Elaftick force of that which is contain'd within the Receiver, and thereby it will open the Valve and make way for it's felf to enter totally into the Receiver as it is continually pufh'd forwards by the defcending Embolus. Since then the Quantities intruded at each ftroke of the Forcer are equal, it manifeftly appears that the Quantities in the Receiver and confequently the degrees of Condenfation do increafe in an Arithmetical Progreffion. Let us now examine by what fteps the Quickfilver in the Gage advances at each ftroke. What I fhall endeavour to prove as to this matter is this: that as the Quickfilver is moved forwards in the Gage upon every fucceffive ftroke of the Forcer, the Spaces at the end of the Gage, which are yet left free from the Quickfilver, do decreafe in a Mufical Progreffion. But in the firft place it may not be amifs to explain in fome meafure the nature of Mufical Progreffions, fince thefe are not generally fo well underftood as thofe which we call Arithmetical and Geometrical Progreffions. In order to do this, I fhall propofe an Inftance which firft gave occafion for the Name. 'Tis a thing well known among Muficians, if three Chords

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or Strings, in all other respects alike, be of different lengths, and those lengths be to each other in proportion as the Numbers 6, 4 and 3, that the Sounds of those Strings will express the principal and most perfect of the Musical Concords, namely, and Eight a Fifth and a Fourth. Thus the Sound of the last will be an Octave to the Sound of the first, and the Sound of the second a Fifth to the Sound of the first, and the Sound of the last a Fourth to the Sound of the second. Hence these Numbers 6, 4 and 3, which express the Proportions of those Musical Strings were said not improperly to be in a Musical Progression. Now it was easy to be observed that these Numbers were reciprocally proportionable to three other Numbers respectively, *viz.* 2, 3 and 4 which were in Arithmetical Progression; and thence it came to pass, that any other Series of Numbers was said to be in a Musical Progression which had the same property of being reciprocally proportionable to a Series of Numbers in Arithmetical Progression. That therefore is a Series of Musical proportionals which is reciprocal to another Series of Arithmetical proportionals. But besides this, you may observe another property belonging to the above mention'd Numbers 6, 4 and 3 *viz.* That the First is to the Third as the difference of the First and Second is to the difference of the Second and Third. And this property does equally belong to all other Numbers, which are reciprocally as a Series in Arithmetical Progression, that is, to all other Numbers which are in a Musical Progression. Hence if any two succeeding Terms be given, the Third may be found by dividing the Product of the First and Second by the difference which arises in Substracting the Second from the double of the First. Thus in the Progression 6, 4, and 3 the Product of the first and second Terms 6 and 4 is 24, and the difference which arises by substracting the second Term ( 4 ) from the double of the first ( 12 ) is 8, and the Quotient which emerges by dividing the Product ( 24 ) by the difference ( 8 ) is 3 the third Term in the Progression required. I shall now go on to shew that the Spaces unpossess'd by the Quicksilver at the end of the Gage do decrease in such a Musical Progression. It must be observed therefore, that the Quicksilver of the Gage is contiguous on one side to the Air within the Receiver and on the other side to the Air which is shut up at the end of the Gage, and the Density of the Air in both Places is equal. For were the Density of the Air in the Receiver greater than the Density of the Air at the end of the Gage ; its Elastick Force would also be greater, and by that Excess of Force the Quicksilver would be moved on further towards the end of the Gage, till the Forces and consequently the Densities became equal. After the same manner if the Density of the Air at the end of the Gage were greater than the Density of the Air within the Receiver ; the Quicksilver would be moved backwards from the end of the Gage, till the Densities became equal. It is manifest therefore that the Densities are equal on both parts

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when the Quickfilver in the Gage is at reft. Therefore fince the Denfity of the Air in the Receiver upon every fucceffive ftroke of the Forcer was increafed in Arithmetical Progreffion, it follows that the Denfity of the Air at the end of the Gage is likewife increafed in the fame Arithmetical Progreffion. But the Space which that Air poffeffes is diminifhed in the fame proportion by which the Denfity is increafed, or in other words, the Spaces are reciprocally as the Denfities: therefore the Spaces are reciprocally as a Series of Terms in Arithmetical Progreffion, which is the fame thing as to fay, the Spaces are in a Mufical Progreffion. And this conclufion we found alfo to agree with our Experiments.

## A Second Enquiry into the State of the Atmofphere.

BEFORE I conclude it may not be amifs in this Place to make our Enquiries once more concerning the State of the Atmofphere, and the different degrees by which the Air is rarified at different Altitudes above the Surface of the Earth. You remember it was proved in the foregoing Week, that the Denfity of the Air was diminifhed in a Geometrical Progreffion as the Altitude of it was increafed in an Arithmetical Progreffion. The truth of that Rule depends upon this Suppofition, that the Gravity of Bodies is the fame at all diftances from the Centre of the Earth. But it has been proved and put beyond difpute by Sir *Ifaac Newton* in his *Principia*, that the Gravity of Bodies is not exactly the fame at all diftances from the Centre, but is diminifhed as the diftance increafes, fo that the Quantity of it is always reciprocally proportionable to the Square of the diftance. From hence it eafily appears that when the Altitude of the Air above the Surface of the Earth is very great and very confiderable in refpect of the Earth's Semidiameter, the Rule which I formerly gave you will be far from being true ; but if the Altitude be fmall and inconfiderable (as the Altitudes of our higheft Mountains muft be confefs'd to be) it will ftill be fufficiently exact, and as fuch it is propofed by Dr. *Halley* in the Philofophical Tranfactions, and by Dr. *Gregory* in his Aftronomy, and generally received by others without any exceptions. However, it may be worth our while to fee what confequences will arife upon the truer Hypothefis which fuppofes (as I faid above ) the Gravity of Bodies to be diminifhed in the fame proportion by which the Square of their diftance from the Centre of the Earth is increafed. In treating of this matter I fear I fhall not be generally underftood, yet I hope I fhall make the thing as eafy as the nature of it will permit. (Fig. 15.) Let *C* reprefent the Centre of the Earth, *CA* its Semidiameter, *AB* a part of its Surface, and let the Line *CAD* be produced up to the extremity of the Atmofphere. In this Line imagine the Points

D,E,F

$D, E, F$ to be placed infinitely near to each other, and take as many other Points $d, e, f$ in such a manner that the distances $dC, eC, fC$ shall be reciprocally proportionable to the distances $DC, EC, FC$ respectively, or in such manner that the distances $dC, eC, fC$ shall be less than the Semidiameter $AC$ in the same proportion by which the respective distances $DC, EC, FC$ are greater than the same Semidiameter: the distances of the lesser Letters from the Centre being diminished in the same proportion by which the distances of the corresponding greater Letters from the Centre are increased. Upon the Points $A, d, e, f$ erect the Perpendiculars $AB, dp, eq, fr$, and suppose the length of these Perpendiculars to be proportionable to the Density of the Air in $A, D, E, F$ respectively so that the Density of the Air at $A$ shall be represented by the Perpendicular $AB$, the Density of the Air at $D$ by the Perpendicular $dp$, the Density at $E$ by the Perpendicular $eq$, and the density at $F$ by $fr$. This being done, I am now to prove, that if the distances $CF, CE, CD$ be taken in a Musical Progression and consequently the distances $Cf, Ce, Cd$, be in an Arithmetical Progression as being reciprocally proportionable to the former distances; the Perpendiculars $fr, eq, dp$, and consequently the Densities of the Air in the places $F, E, D$ which are analogous to the Perpendiculars will be in a Geometrical Progression. In the first place then because the distances of the lesser Letters from the Centre are reciprocally as the distances of their correspondent greater Letters from the same, it is manifest that $Cd$ is to $Ce$ as $CE$ is to $CD$, and consequently the difference of $Cd$ and $Ce$ is to the difference of $CE$ and $CD$ as $Ce$ to $CD$, or (because the Points $E$ and $D$ are supposed to be infinitely near to each other) as $Ce$ to $CE$ or (because $Ce$ is less than $CA$ in the same proportion by which $CE$ is greater than $CA$, and consequently $Ce$, $CA$ and $CE$ are continual proportionals) as $CAq$ is to $CEq$. It is evident then, that $de$ (the difference of $Cd$ and $Ce$) is to $DE$ ( the difference of $CE$ and $CD$) as $CAq$ is to $CEq$. Therefore if the distance $CE$ remain unaltered and consequently the proportion of $CAq$ to $CEq$ remain unaltered, the proportion of $de$ to $DE$ will also remain unaltered, and consequently $de$ will be as $DE$, that is, $de$ will be increased and diminished in the same proportion with $DE$. But if $DE$ remain unaltered, because it is always greater than $de$ in the proportion by which $CEq$ is greater than $CAq$, it follows that $de$ must necessarily be diminish'd in the same proportion by which $CEq$ is increased, and increased in the same proportion, by which $CEq$ is diminished, or in other words it must always of necessity be reciprocally as $CEq$; whence it follows, that if neither $DE$ nor $CE$ remain unaltered, $de$ will be as $DE$ directly and as $CEq$ reciprocally. But the Bulk of Air between the places $D$ and $E$ is as $DE$, and the gravity of the same is reciprocally as the Square of $CE$ it's distance from the Centre: therefore $de$ is as the Bulk and Gravitation together of the same, and consequently

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ly since *eq* is as its Density the Product of *de* and *eq* or the Area *deqp*
will be as the Product of its Density, Bulk, and Gravitation, that is, as
its Force to compress the inferior Air. And the sum of all such Areas
below *dp* will be as the sum of such Forces of all the Air above *D*, that
is, as *dp* the Density of the Air at *D*, for you know the Density of the
Air is always as the Force which compresses it. Since the Perpendicular
*dp* is as the sum of all the little Areas below its self and the Perpendicular
*eq*, for the same reason, is as the sum of all below its self: it follows, that
the difference of *eq* and *dp* is as the difference of those Sums, which
difference is the Area *eqpd*. Thus far then we have proceeded, we have
found that the difference of the Perpendiculars *eq* and *dp* is as the Area
*eqpd* comprehended by those Perpendiculars. Let us now suppose the
distances *CF*, *CE*, *CD*, and so on, to be taken in a Musical Progression
and then (as was said above) the distances *Cf*, *Ce*, *Cd*, and so on, will
be in an Arithmetical Progression, and therefore all the Intervals *de*, *ef*
will be equal and consequently the Areas *eqpd* which have those equal
Intervals for their Bases will be as their Altitudes *eq*. Hence the diffe-
rence of *eq* and *dp* which was as the Area *eqpd* will be as *eq* and conse-
quently *dp* will be as *eq*. In other words, the two Perpendiculars which
terminate the little Area included between them do every where bear the
same given proportion to each other. Therefore the proportion of *fr* to
*eq* is the same with the proportion of *eq* to *dp*, and consequently the
Perpendiculars *fr*, *eq*, *dp*, and so on, are in a Geometrical Progression.
But these Perpendiculars express the Densities of the Air at the places *F*,
*E*, *D*, and so onwards. Therefore those Densities are also in a Geometri-
cal Progression, which was the thing to be proved. To proceed further:
since *Cd* is to *CA* as *CA* is to *CD*, it follows that *Ad* is to *AD* as
*CA* to *CD*, or in other words that *Ad* is less than *AD* in the same pro-
portion by which the Semidiameter of the Earth is less than the distance
of the Point *D* from the Centre; consequently to find the length of *Ad*
we must diminish the Altitude *AD* in the proportion of the Semedia-
meter of the Earth to the sum of the Semidiameter and the Altitude, for
which reason I shall call *Ad* the diminish'd Altitude of the Point *D*, and
upon the same account *Ae* may be call'd the diminish'd Altitude of the
Point *E*, and *Af* the diminish'd Altitude of the Point *F*; and so if *h*
be the Point which corresponds as above to the Point *H*, *Ah* will be the
diminish'd Altitude of the Point *H*. Now 'tis easy to observe that as
the distances *Cd*, *Ce*, *Cf* are in Arithmetical Progression, so are also the
diminish'd Altitudes *Ad*, *Ae*, *Af*. And from hence there arises this
Theorem. That if the diminish'd Altitudes be taken in Arithmetical Pro-
gression the Densities of the Air will be in a Geometrical Progression.
Therefore if the Rarity of the Air at any one Altitude, suppose at *H*,
be known, you may easily enough find its Rarity at any other Altitude

suppose

suppose at $D$. For as the diminish'd Altitude of the Point $H$ is to the diminish'd Altitude of the Point $D$, so will be the Logarithm of the Air's Rarity at $H$ which is supposed to be known, to the Logarithm of the Airs Rarity at $D$ which was to be found. The whole difficulty of the business is therefore reduc'd to this: to find the Rarity of the Air at some one Altitude as at $H$. This may be done as I formerly shew'd you by carrying the Barometer to the top of some very high Mountain and observing the descent of the Quicksilver. Such were the Experiments made upon the *Puy de Domme* in *France*, and *Snowden Hill* in *Wales*, which I made use of the last Week when I discours'd of this Subject. But the Method I shall now describe to you is more expeditious and depends upon the Experiment which we made this Day. It appeares ( as I said ) by many such Experiments compar'd together, that the weight of Air is to the weight of Water as 1 to about 850. Therefore a Column of Air whose height is 850 Inches or 70 Feet and 10 Inches will be equal in weight to a Column of Water upon the same Basis whose height is 1 Inch. Let us suppose that $AH$ the height of the Point $H$ above the Surface of the Earth is 70 Feet and 10 Inches; then because the Standard height of Water in the Pascalian Tube is 34 Feet or 408 Inches, and this height of Water is a Ballance to the Pressure of the whole Atmosphere upon the Surface of the Earth, 'tis manifest that the weight of the whole Column of Air, which is superiour to the Point $A$, is equal to the weight of a Column of Water upon the same Basis whose height is 34 Feet or 408 Inches. Take from the weight of the whole Column of Air the weight of that part of the Column which reaches from $A$ up to $H$, and which was shewn to be equal to one Inch of Water; and the weight of the remaining part of the Column which is above the the Point $H$ will be equal to the weight of 407 Inches of Water. Therefore the Force with which the Air at $A$ is compress'd is to the Force with which the Air at $H$ is compress'd as 408 to 407, and the Rarity of the Air at $H$ is to the Rarity of the Air at $A$ in the same proportion. You may perceive that this Method supposes the Air to be of the same Density in every part of the Space $AH$, which is not exactly true; but in so small an Altitude as that of 70 Feet the Error is altogether insensible. However if you have a mind to proceed with the utmost accuracy, you may do so, by making the Altitude as small as you please.